

Implementing universal quantum gates in coupled cavities

Dimitris G. Angelakis and Alastair Kay

*Centre for Quantum Computation, Department of Applied Mathematics and Theoretical Physics,
University of Cambridge, Wilberforce Road, CB3 0WA, UK*

We study a linear array of coupled cavities interacting with two level systems and show how to construct stable, individually addressable, qubits in this system from the long-lived atom-photon excitations (polaritons) at each site. We derive the system dynamics and show that is described by an XY Hamiltonian. We proceed by showing how to implement non-local quantum gates and show that combined with the inherent ability for individual addressing, universal quantum computation is possible in this system. We finally discuss the prospects for experimental implementation using various technologies involving dopants as atoms, quantum dots or Cooper pair boxes.

Introduction

A continuing problem in quantum information is to find physical systems in which universal quantum computation can be implemented. Existing proposals involve a wide variety physical systems including linear optics, ion traps, NMR, quantum dots, superconductors, neutral atoms in optical lattices, and flying atom schemes. Most of these schemes are able to achieve a high degree of control over a small number of systems, but are limited in their scalability. In contrast to this, optical lattices have been extremely successful in creating coherent states of a large number of atoms, but suffer from the inability (or extreme difficulty) of performing operations on individual atoms[1].

On the other hand, there have recently been theoretical and experimental breakthroughs into the possibility of directly coupling high-Q cavities together, and in achieving strong coupling between the cavity mode and an embedded two-level system. A variety of technologies have been employed, including fiber coupled micro-toroidal cavities interacting with atoms [2, 3], arrays of defects in photonic band gap materials (PBGs) [4, 5, 6] and superconducting qubits coupled through microwave stripline resonators [7]. This has prompted proposals for the implementation of optical quantum computing [8], the production of entangled photons [9] and the realization of Mott insulating and superfluid phases [10, 11, 12]. Here we consider the use of such arrays for the realization of universal quantum computation.

System

We start by describing the system and demonstrating how the complex energy level structure of hybrid light-matter excitations (polaritons) can be reduced to an effective two-level system. If we consider a chain of N cavities (which is located in a defect) in 1D, then photons can propagate between the cavities, and this hopping mechanism induces a coupling between the cavities. A realization of this has been studied in structures known as a coupled resonator optical waveguides (CROW) or couple cavity waveguides (CCW) in photonic crystals, where it was shown that light propagation is characterized by small dispersion, very small group velocities and low losses. The dispersion relation can be derived in a simple way by applying the tight binding formalism from the electronic band theory. The eigenmode for the extended structure is given by

$$\mathbf{E}(r, k) = \sum_n e^{-ink\Lambda/N} E_n(r) \quad (1)$$

where $E_n(r) = E_m(r - (n - m)\Lambda)$ describe the localized ground state modes for each cavity (Wannier functions) and Λ is the distance between the defects and the summation over n includes all the cavities. If we assume that $\Lambda = 1$ for simplicity, then the dispersion relation is given by

$$\omega(k) = \omega_d[1 + A \cos(2\pi k/N)], \quad (2)$$

where A is the tight binding parameter which depends on the geometry and Λ , and ω_d is the frequency of an individual defect.

The usual way to quantize a system is through its eigenmodes. The Hamiltonian corresponding to the above dispersion relation is given by

$$H = \omega_d \sum_m b_m^\dagger b_m [1 + A \cos(2\pi k/N)] \quad (3)$$

where $b_k^\dagger(b_k)$ are the creation (annihilation) operators of photons occupying the extended eigenmode. We could equally well describe the system dynamics using the operators of the localized eigenmode (Wannier functions), $a_k^\dagger(a_k)$. These describe the creation (annihilation) of a photon in the localized defect mode k . It's straightforward to see that the transformation connecting the two bases is given by

$$b_k = \frac{1}{N} \sum_{m=-N/2}^{N/2} a_m e^{-2\pi i k m / N}. \quad (4)$$

In the localized mode basis, the Hamiltonian is given by

$$H = \sum_{k=1}^N \omega_d a_k^\dagger a_k + \sum_{k=1}^N A(a_k^\dagger a_{k+1} + H.C.) \quad (5)$$

Now assume that the cavities are doped with two level systems (atoms or a quantum dots). We shall denote the two energy levels by g_N and e_N corresponding to the ground and excited states of a dopant placed at defect N . The Hamiltonian describing the system is

$$H = \sum_{k=1}^N H_k^{free} + H_k^{int} + H_k^{hop}, \quad (6)$$

where H^{free} is the Hamiltonian for the free light and atom parts, H^{int} the Hamiltonian describing the internal coupling of the photon and atom in a specific defect and H^{hop} describes the light hopping between defects.

$$H^{free} = \omega_d |1_k\rangle\langle 1_k| + \omega_0 |e_k\rangle\langle e_k| \quad (7)$$

$$H^{int} = g(|1_k g_k\rangle\langle 0_k e_k| + H.C.) \quad (8)$$

$$H^{hop} = A(|1_k 0_{k+1}\rangle\langle 0_k 1_{k+1}| + |0_k 1_{k+1}\rangle\langle 1_k 0_{k+1}|) \quad (9)$$

ω_d and A are the photon frequencies and hopping rates respectively and g is the light-atom coupling strength. The $H^{free} + H^{int}$ component of the Hamiltonian can be diagonalized in a basis of combined photonic and atomic excitations, called *polaritons*. These polaritons are defined by creation operators $P_k^{(\pm, n)\dagger} = |n\pm\rangle_k \langle g, 0|_k$, where the polaritons of the k th atom-cavity system are given by $|n\pm\rangle_k = (|g, n\rangle_k \pm |e, n-1\rangle_k) / \sqrt{2}$ with energies $E_n^\pm = n\omega_d \pm g\sqrt{n}$, and $|n\rangle_k$ denotes the n -photon Fock state. As has been shown elsewhere, a polaritonic Mott phase exists in this system where a maximum of one excitation per site is allowed [10]. This originates from the repulsion due to the photon blockade effect [13, 14]. In this Mott phase, the system's Hamiltonian can be written in the interaction picture as

$$H_I = A \sum_{k=1}^{N-1} P_k^\dagger P_{k+1} + P_k P_{k+1}^\dagger, \quad (10)$$

where $P_k^\dagger = P_k^{(-, 1)\dagger}$. As double or more occupancy of the sites is prohibited, one can identify P_k^\dagger with $\sigma_k^+ = \sigma_k^x + i\sigma_k^y$, where σ_k^x and σ_k^y are the standard Pauli operators. The system's Hamiltonian then becomes the standard XY model of interacting spin qubits with spin up/down corresponding to the presence/absence of a polariton.

$$H_I = A \sum_{k=1}^{N-1} \sigma_k^x \sigma_{k+1}^x + \sigma_k^y \sigma_{k+1}^y. \quad (11)$$

Some applications of XY spin chains in quantum information processing can thus been implemented in this system [15]

Implementing control phase gates

Now that we have a system of effective two-level atoms, we can treat these as qubits and try to find how to implement universal quantum computation with them. By construction, single atoms can undergo local rotations by the application of laser fields, and the lattice sites can be spaced sufficiently far apart that lasers can individually

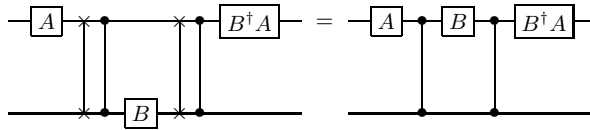


FIG. 1: The two-qubit gate we realise, in conjunction with single-qubit rotations, is sufficient to create a controlled- U gate, and is hence universal.

address each site. Thus, all we need to demonstrate is an entangling gate between pairs of qubits. In order to achieve this, we will use the natural system dynamics of the XY Hamiltonian which we have just demonstrated.

For a chain of 3 qubits, the Hamiltonian is

$$H = A(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y + \sigma_2^x \sigma_3^x + \sigma_2^y \sigma_3^y)$$

and one can readily verify the following action on four of the states,

$$\begin{aligned} e^{-iH\pi/(2\sqrt{2}A)} |000\rangle &= |000\rangle \\ e^{-iH\pi/(2\sqrt{2}A)} |001\rangle &= -|100\rangle \\ e^{-iH\pi/(2\sqrt{2}A)} |100\rangle &= -|001\rangle \\ e^{-iH\pi/(2\sqrt{2}A)} |101\rangle &= -|101\rangle. \end{aligned}$$

This can be assigned the interpretation that if the central qubit is in the state $|0\rangle$, the state of the other two qubits is swapped, and both local Z -rotations and a controlled-phase gate are applied to them. Of central importance among these is the controlled-phase gate, precisely the entangling gate that we require. If the central qubit is in the $|1\rangle$ state, then the same effect is realised without the local Z -rotations [17]. Thus, we do not in fact need to know the state of the central qubit, we simply need to measure it after the evolution time of $\pi/(2\sqrt{2}A)$ and by applying a measurement on the ‘mediator’ qubit (the middle of the chain of three) in the σ^z basis, a nonlocal gate results between the two extremal qubits [18]. Depending on the measurement result, $|0\rangle$ or $|1\rangle$, the operation performed between the two computational qubits was either $\text{SWAP} \cdot (\sigma^z \otimes \sigma^z) \cdot CP$ or $\text{SWAP} \cdot CP$ respectively. The local σ^z rotations are readily compensated for, and the effect of the SWAP gates can be tracked to make sure the correct qubits interact when we want them to. This non-local gate combined with the ability to perform individual rotation of the polaritonic qubits allow for universal quantum computation in this system (the cavities can be well separated in contrast to optical lattices implementations for example). This can be seen for example by considering the construction in Fig. 1 which allows, with two applications of this gate, the construction of any controlled- U gate, where $U = A^\dagger B^\dagger \sigma^z B \sigma^z A$. Hence we can separate the action of the SWAP and CP .

In order to implement this scheme experimentally, dissipation due to spontaneous emission and cavity leakage need to be taken into account. As previously mentioned, there are three primary candidate technologies; fiber coupled micro-toroidal cavities [2, 3], arrays of defects in PBGs [4, 5, 6] and superconducting qubits coupled through microwave stripline resonators [7]. In order to achieve the required limit of no more than one excitation per site [10], the ratio between the internal atom-photon coupling and the hopping of photons down the chain should be of the order of $g/A \sim 10^2 - 10^4$ (A can be tuned while fabricating the array by adjusting the distance between the cavities and g depends on the type of the dopant). In addition, the cavity/atomic frequencies to internal coupling ratio should be $\omega_d, \omega_0 \sim 10^4 g, 10^5 g$ and the losses should also be small, $g/\max(\kappa, \gamma) \sim 10^3$, where κ and γ are cavity and atom/other qubit decay rates. The polaritonic states under consideration are essentially unaffected by decay for a time $10/A$ (10ns for the toroidal case and 100ns for microwave stripline resonators). The required parameter values are currently on the verge of being realised in both toroidal microcavity systems with atoms and stripline microwave resonators coupled to superconducting qubits, but further progress is needed. Arrays of defects in PBGs remain one or two orders of magnitude away, but recent developments, and the integrability of these devices with optoelectronics, make this technology very promising as well. In all implementations the cavity systems are well separated by many times the corresponding wavelength of any local field that needs to be applied in the system for the measurement process.

Conclusions

In this paper, we have shown how universal quantum computation could be realized in a coupled array of individually addressable atom-cavity systems, where the qubits are given by mixed light-matter excitations in each cavity site.

While single-qubit operations can be locally achieved, the only available interaction between qubits is due to the natural system Hamiltonian. We show how to manipulate this to give a controlled-phase gate between pairs of qubits. This combined with the inherent ability of the system for individual addressing allows for universal quantum computation. We have discussed possible architectures for implementing these ideas using photonic crystals, toroidal microcavities and superconducting qubits and point out their feasibility and scalability with current or near-future technology.

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